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Quasi stable solitons generated by landscape

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Abstract

Based on the landscape idea we consider solitons formation during inflation. We consider special case of scalar field dynamics and discuss characteristics of non-trivial field configurations, obtained in our approach.

Keywords: solitons, kinks, inflation, scalar fields, cosmology.

1. Introduction

Field theory provides us with well-known stable non-trivial field solutions – solitons, kinks [1], monopoles [2,3]. Usually their stability is defined by the topology of the system. In this work we show that the inflationary process can give rise to quasi stable solitons.

After inflation such classical solutions can reveal themselves as enhanced energy density, or, by decaying, as local heat sources.

It is possible in the framework of Multi-Stream Inflation [4,5] and two-field inflation. This approach is based on the landscape idea [6] which explains the existence of numerous local extrema. These extrema lead to the formation of non-trivial classical field solutions. In the next section we give numerical analysis.

In [4] the authors consider the potential $V(\phi,\chi)$ as illustrated in Fig. 1. In the beginning, the ϕ direction is the inflationary direction. At the point ϕ_1 the inflationary trajectory spontaneously breaks into two paths, namely A and B in Fig. 1. The whole picture looks like a stream which splits and flows along both sides of a hill. So the authors call this scenario "multi-stream" inflation. As inflation continues, the trajectories A and B may either recombine into a single trajectory or not. Which was not considered in [4] is the case when the bump of the potential $V(\phi,\chi)$ is high enough for the trajectory to "catch on" it. In the next section we are going to discuss such a case and it's possible sequences.

Authors also claim that in the case when the paths A and B have slightly different potential energy, multi-stream inflation has several interesting differences from the usual inflation model. They calculate the density perturbation

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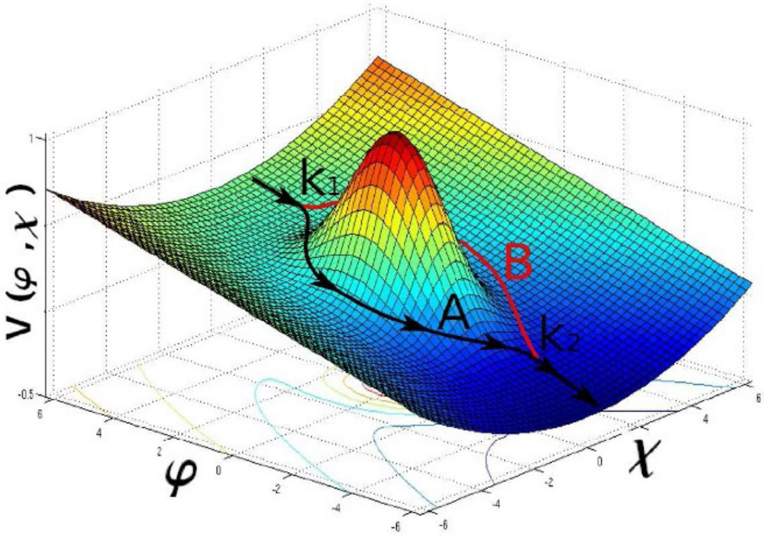

Fig. 1. The illustration of multi-stream inflation. The picture was taken from [4].

and non-Gaussianity in the framework of multi-stream inflation and discuss features in the CMB power spectrum and hemispherical power asymmetry as consequences of multi-stream inflationary approach.

Let us now analyze the physical mechanism of solitons formation in our approach.

It is widely known that fields fluctuations during inflation lead to inhomogeneous fields distribution. Assume that we can define fields ϕ and χ in the way that potential $V(\phi,\chi)$ has a minimum at ϕ_0 and a local maximum at ϕ_1. Let us also assume that we have a space region filled with fields ϕ_{in} and χ_{in}, and suppose that $\phi_0 < \phi_{in} < \phi_1$ and the interval for χ_{in} contains zero value of this field. In this case field in the whole space region will tend to the minimum of the potential. But if we have the initial conditions in the form $\phi_0 < \phi_1 < \phi_{in}$ then the space region will split up into two – on the right and on the left of the locus P, containing values of the field ϕ_1 at a maximum. Line connecting two points on different sides of the point ϕ_1 contains all values of χ. Boundary points and points P lie in causally independent regions. That's why fields motions towards the minimum happen independently at the boundary points. The gradient field has a direction towards the minimum (ϕ_0,χ_0) at all points except the region near the maximum (ϕ_1,χ_1), where it has a direction from the maximum. It means that we have an equilibrium point (ϕ_e,χ_e), where the gradient field is equal to zero (saddle point).

Thus we have a stationary solution $\phi(x),\chi(x)$ for classical equations. This solution possesses two equilibrium points – the saddle point (ϕ_e,χ_e) and the minimum of the potential at $(\phi_0,0)$. It forms a closed trajectory $\chi(\phi)$, surrounding the maximum of the potential, which prevents this trajectory from collapsing to a point.

2. Fields motion after inflation

Let us consider the potential $V(\varphi,\chi)$ in the following form:

$$V(\varphi,\chi) = a \cdot e^{\left(-b(\varphi-\varphi_1)^2-c(\chi-\chi_1)^2\right)} + \epsilon_1 \cdot (\varphi-\varphi_2)^2 + \epsilon_2 \cdot (\chi-\chi_2)^2, \tag{1}$$

with $a, b, c, \varphi_1, \varphi_2, \chi_1, \chi_2, \epsilon_1, \epsilon_2$ being constants.

With some set of parameters this potential has an interesting shape, presented in Fig. 2: it possesses an absolute minimum at $(\varphi = 0, \chi = 6)$ and also has a bump at $(\varphi = 0, \chi = 0)$. Characteristics of the bump are crucial for the dynamics of the fields ϕ and χ if one considers trajectories $\chi(\varphi)$, surrounding the bump.

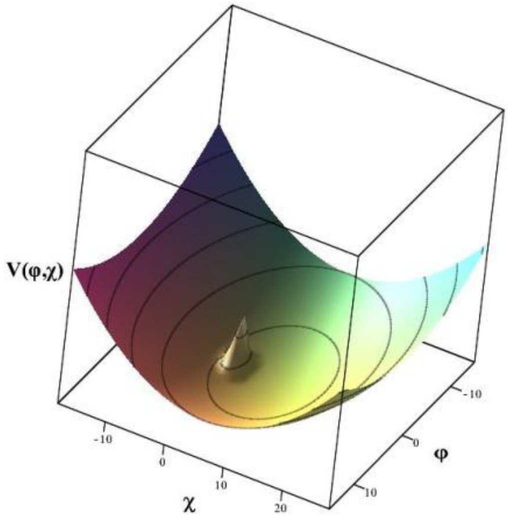

Fig. 2. Potential (1) with the following parameters values: $a = 25, b = 1, c = 0.5, \phi_1 = \chi_1 = \chi_2 = 0, \phi_2 = 6, \epsilon_1 = \epsilon_2 = 0.08$.

To study the dynamics of fields φ and χ let us consider the equations of motion. To be more specific we assume that φ and χ are functions of time t and single space coordinate x:

$$\begin{aligned} &\partial_t^2\varphi - \partial_x^2\varphi - \partial_\varphi V(\phi,\chi) + 3H\partial_t\varphi = 0, \\ &\partial_t^2\chi - \partial_x^2\chi - \partial_\chi V(\phi,\chi) + 3H\partial_t\chi = 0, \end{aligned} \tag{2}$$

where H is the Hubble parameter (hereinafter we assume that $H = 10^{-1}H_{infl} \approx 10^{12}$ GeV).

It appears that the analytical solution of (2) is of great challenge, so we will be interested only in numerical solution.

Let us pass to the polar coordinates for simplisity:

$$\begin{aligned} &\varphi = r \cdot sin(\theta), \\ &\chi = r \cdot cos(\theta). \end{aligned} \tag{3}$$

Then we can write down initial and boundary conditions for the system (2) in terms of r and θ in the following way:

$$\begin{aligned} &r(x,0) = r_0 + e^{-x^2}, \\ &\theta(x,0) = \pi \cdot tanh(x) + \pi + \theta_0, \\ &r(\pm\infty,t) = r_0, \\ &\theta(\pm\infty,t) = \theta_0, \end{aligned} \tag{4}$$

with (r_0, θ_0) being coordinates of the absolute minimum $(\varphi = 0, \chi = 6)$ of the potential (1).

Conditions (4) give us closed trajectories in (χ, φ) phase space, surrounding the bump of the potential in $(\varphi = 0, \chi = 0)$.

Solving (2) together with (4) provides us with the dynamics of the fields $\varphi(x,t)$ and $\chi(x,t)$. The behaviour of the field χ is more representative – it is shown in Fig. 3 for three different moments of time: the dashed line is the initial distribution $\chi(x, t = 0)$ and the solid one corresponds to $\chi(x, t \to +\infty)$ (dash-dotted line corresponds to some intermediate moment of time). One can see that field χ forms some kind of stable configuration possessing nonzero energy. The same solution $\chi(x)$ but for different set of parameters is shown in Fig. 4.

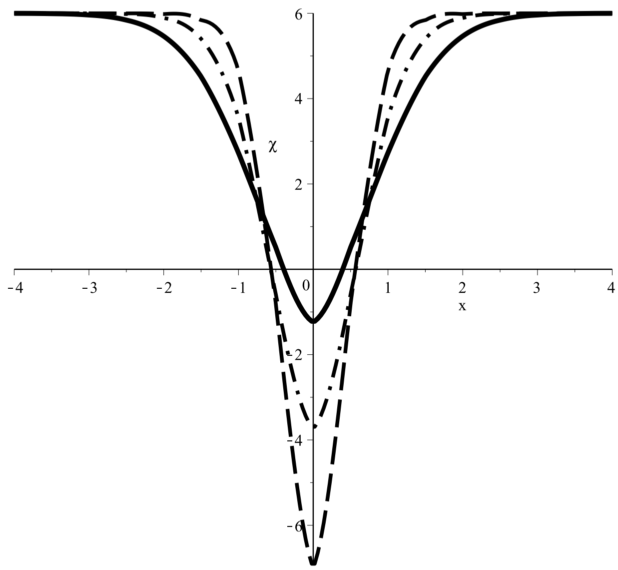

Fig. 3. Numerical solution $\chi(x)$ with some set of parameters (see caption in Fig. 2).The scalar field dynamics for different moments of time: the dashed line is the initial distribution $\chi(x, t = 0)$, the solid one corresponds to $\chi(x, t \to +\infty)$ and dash-dotted – to some intermediate moment of time

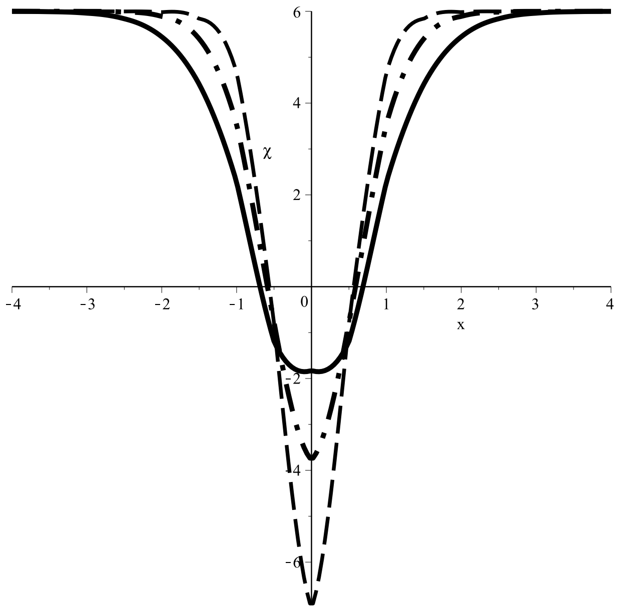

Fig. 4. Numerical solution $\chi(x)$ with the same set of parameters as in Fig. 3, but with $a = 50$: the dashed line is the initial distribution $\chi(x, t = 0)$, the solid one corresponds to $\chi(x, t \to +\infty)$ and dash-dotted – to some intermediate moment of time.

3. Energy density

Let us also estimate the energy density of this system of two fields in the final state (corresponding to $t \to +\infty$):

$$\epsilon(x) = (\partial_x\varphi)^2 + (\partial_x\chi)^2 + V(\varphi, \chi), \tag{5}$$

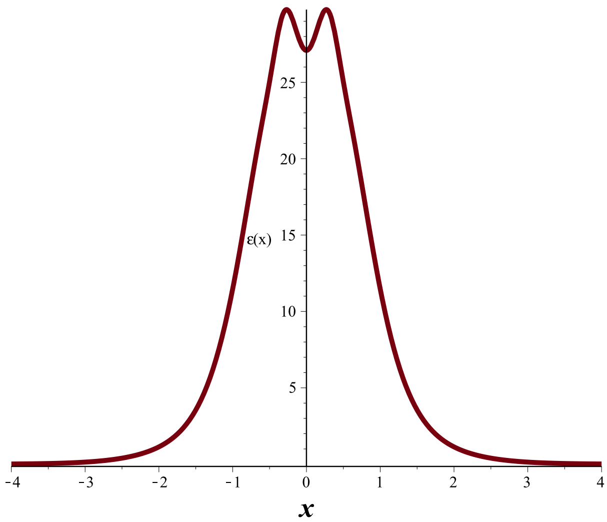

Fig. 5. Energy density (5) of the final state of the system of two fields φ and χ.

and the surface energy density

$$W = \int_{-\infty}^{+\infty} \epsilon(x) dx \simeq 6 \cdot 10^{-14} M_{pl}^{3} \simeq 3 \cdot 10^{44} \left(\frac{g}{cm^2}\right) \tag{6}$$

where M_{pl} is the Planck mass.

Energy density dependence on x is shown in Fig. 5. One can see that the energy density profile has an interesting shape, which is the consequence of the shape of the scalar fields potential energy (1).

4. Discussion

We showed that the inflationary process can lead to the formation of quasi stable solitons. Based on the idea of the landscape we considered special case of the fields dynamics: scalar fields trajectory can "catch on" the bump of the potential, thus forming non-trivial fields configuration. By calculating its surface energy density (6) one can conclude that such fields configurations can be observed as regions of enhanced energy density or as local heat sources.

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